

## Preface

This volume contains the Proceedings of the 41st International Conference on Mathematical Foundations of Programming Semantics (MFPS XLI), which was held from 18th to 20th June 2025 at the University of Strathclyde in Glasgow, Scotland. The conference was colocated with the 11th Conference on Algebra and Coalgebra in Computer Science (CALCO), which took place from 16th to 18th June.

We thank the local organizers, Bob Atkey, Dilsat Bilal Yuksel, Clemens Kupke, and the Mathematically Structured Programming Group for the smooth running of the event, and for ensuring a successful and above all enjoyable colocation of MFPS and CALCO. We also thank the MFPS Organizers (Steering Committee), in particular Michael Mislove, for their guidance and advice.

MFPS conferences are dedicated to the areas of mathematics, logic, and computer science that are related to models of computation in general, and to semantics of programming languages in particular. This is a forum where researchers in mathematics and computer science can meet and exchange ideas. Topics include, but are not limited to, the following: bio-computation, concurrent qualitative and quantitative distributed systems, process calculi, probabilistic systems, constructive mathematics, domain theory and categorical models, formal languages, formal methods, game semantics, lambda calculus, programming-language theory, quantum computation, security, topological models, logic, type systems and type theory.

MFPS 2025 continued the tradition of having an exciting group of invited speakers, both plenary and in special sessions. We enjoyed listening to Gordon Plotkin, who gave a joint plenary talk with CALCO, and to Chris Heunen, Delia Kesner, and Jurriaan Rot, who all gave MFPS plenary talks. MFPS and CALCO held a joint special session on Quantitative Semantics, organized by Ugo Dal Lago. Additionally, MFPS held special sessions on Mathematics of Natural Language, organized by Mehrnoosh Sadrzadeh, and on Types and the Extraction of Correct Programs, organized by Ulrich Berger.

The conference received 28 submissions in total. They were received from Argentina, Australia, Austria, Estonia, France, Italy, Japan, The Netherlands, Poland, Romania, Slovenia, the United Kingdom, and the United States. The program committee accepted 18 submissions for presentation. We are very grateful to the program committee members and external reviewers for their exceptional work over an extremely tight time schedule. The accepted papers are published in this volume (in alphabetical order of the names of first authors). We hope you will enjoy reading the papers in this volume as much as we enjoyed listening to the talks.

*Clemens Kupke and Stefan Milius  
December 2025*

## Conference Organization

### Program Committee

Sandra Alves	<i>University of Porto, Portugal</i>
Giorgio Bacci	<i>Aalborg University, Denmark</i>
Lars Birkedal	<i>University of Aarhus, Denmark</i>
Florence Clerc	<i>Heriot-Watt University, Edinburgh, Scotland</i>
Robin Cockett	<i>Univeristy of Calgary, Canada</i>
Claudia Faggian	<i>IRIF Paris, France</i>
Giulio Guerrieri	<i>University of Sussex, UK</i>
Shin-ya Katsumata	<i>NII, Tokyo, Japan</i>
S. Krishna	<i>IIT Bombay, India</i>
Clemens Kupke	<i>University of Strathclyde, Scotland (co-chair)</i>
Elena di Lavore	<i>University of Pisa, Italy</i>
Paul Levy	<i>University of Birmingham, UK</i>
Isabella Mastroeni	<i>University of Verona, Italy</i>
Stefan Milius	<i>FAU Erlangen-Nürnberg, Germany (co-chair)</i>
Daniele Nantes-Sobrinho	<i>Imperial College London, UK</i>
Fredrik Nordvall Forsberg	<i>University of Strathclyde, Scotland</i>
Catuscia Palamidessi	<i>Inria, France</i>
Prakash Panangaden	<i>McGill University, Montreal, Canada</i>
Simon Perdrix	<i>Inria LORIA, France</i>
Elaine Pimentel	<i>University College London, UK</i>
Alex Simpson	<i>University of Ljubljana, Slovenia</i>
Sam Staton	<i>University of Oxford, UK</i>
Dario Stein	<i>Radboud University Nijmegen, The Netherlands</i>
Qiyi Tang	<i>University of Liverpool, UK</i>
Stelios Tsampas	<i>University of Southern Denmark, Odense, Denmark</i>

**Organizers (Steering Committee)**

Andrej Bauer, *University of Ljubljana, Slovenia*

Michael Mislove, *Tulane University, USA*

Lars Birkedal, *Aarhus University, Denmark*

Joël Ouaknine, *MPI SWS, Germany*

Stephen Brookes, *Carnegie Mellon University, USA*

Valeria de Paiva, *Topos Institute, USA*

Achim Jung, *University of Birmingham, UK*

Prakash Panangaden, *McGill University, Canada*

Marie Kerjean, *LIPN, Paris*

Alexandra Silva, *Cornell University, USA*

Paul Levy, *University of Birmingham*

Alex Simpson, *University of Ljubljana, Slovenia*

Catherine Meadows, *Naval Research Laboratory, USA*

Sam Staton, *University of Oxford, UK*

**External Reviewers**

Benedict Bunting, Pierre Clairambault, Robin Cockett, Jonas Forster, Sergey Goncharov, Tomáš Gonda, Justin Hsu, Benjamin Lucien Kaminski, Nicolai Kraus, Denis Kuperberg, Quang Loc Le, Giulio Manzonetto, Claudio Antares Mezzina, David Jaz Myers, G. Michele Pinna, Wojciech Różowski, Alessio Santamaria, Ralph Sarkis, Zev Shirazi, Deivid Vale, Fabian Wiesner, Noam Zilberstein.

# Conference Program Highlights

## Invited Plenary Talks

**Gordon Plotkin** (University of Edinburgh) *Differentiating Functional Reactive Programming*  
(joint with CALCO)

**Chris Heunen** (University of Edinburgh) *Free Quantum Computing*

Quantum computing improves substantially on known classical algorithms for various important problems, but the nature of the relationship between quantum and classical computing is not yet fully understood. This relationship can be clarified by free models, that add to classical computing just enough physical principles to represent quantum computing and no more. I will discuss an axiomatisation of quantum computing that replaces the standard continuous postulates with a small number of discrete equations, as well as a free model that replaces the standard linear-algebraic model with a category-theoretical one. The axioms and model are based on reversible classical computing, and isolate quantum advantage in the ability to take certain well-behaved square roots. The free model may be interpreted as a programming language for quantum computers, that has the same expressivity and computational universality as the standard model, but additionally allows automated verification and reasoning.

**Delia Kesner** (IRIF – Université Paris Cité) *Observational Equivalence by Means of Intersection Types*

Observational equivalence (also known as contextual equivalence) identifies two programs as equivalent when no context can distinguish between them based on their observable behavior. More precisely, two programs  $A$  and  $B$  are observationally equivalent if, for every possible context  $C$ , the observable outcomes of  $C[A]$  and  $C[B]$  are indistinguishable.

This talk revisits classical results on observational equivalence in the call-by-name setting and presents new contributions in the call-by-value paradigm. The approach relies on intersection types, a powerful and flexible typing discipline that allows a single program to be assigned multiple types simultaneously, each one reflecting a different aspect of the term’s behavior along an evaluation path.

**Jurriaan Rot** (Radboud University Nijmegen) *Behavioural Apartness of State-Based Systems*

Behavioural equivalences such as bisimilarity have been widely studied, for a variety of state-based systems such as transition systems and automata. In this talk I will focus on the formal dual of bisimilarity, referred to as apartness by Geuvers and Jacobs (2022), inspired by constructive mathematics. Where bisimilarity can be viewed as a coinductive characterisation of behavioural equivalence, apartness instead is an inductive characterisation of differences in behaviour of systems. Indeed, this form of apartness allows the derivation of explicit proofs of differences.

In the talk, I will describe the basic notions of apartness of state-based systems through several examples, discuss connections to distinguishing formulas in modal logic and to bisimilarity games, and finally discuss ongoing work on quantitative notions of behavioural apartness (the latter is joint work with Ruben Turkenburg, Harsh Beohar, Franck van Breugel and Clemens Kupke).

## Special Sessions

### Quantitative Semantics (joint with CALCO)

This session was organized by Ugo Dal Lago and contained the following invited talks:

**Daniela Petrişan** (IRIF – Université Paris Cité) *Up-to Techniques for Behavioural Metrics*

When analyzing the behavior of state-based systems that incorporate quantitative information, such as probabilities, bisimulations are usually replaced by behavioural metrics. In this talk, I first explain how behavioural metrics between systems modeled as coalgebras can be understood as coinductive predicates in a fibration. I then focus on how to compute such metrics more efficiently, using the so called up-to techniques. These provide a well-known method for enhancing coinductive proofs of behavioural equivalences.

I discuss an extension of up-to techniques for behavioural metrics and provide abstract results to prove their soundness in a compositional way.

**Radu Mardare** (Heriot-Watt University, Edinburgh) *Up-to Techniques for Behavioural Metrics*

This talk is meant to be a tutorial on the current research in the field of Quantitative Equational Logic and Quantitative Algebras. The intention is to cover the main concepts and constructions, the relevant examples, the varieties and quasivarieties theorems, as well as the extensions of the to fix-point theories. The talk will also describe recent directions and future challenges.

**Paolo Pistone** (Université Claude Bernard Lyon 1) *On Generalized Metrics for Higher-Order Languages*

The pervasive role of probabilistic and approximate programming paradigms has motivated the look for semantic approaches not focused on the familiar notion of program equivalence (do two programs compute the same function?), but rather on some notion of program similarity (are the values produced by the two programs close enough?). In these approaches programs are typically interpreted in categories of metric spaces and Lipschitz-continuous functions. However, interpreting higher-order languages in this setting may be challenging. In fact, while the category  $\text{Met}$  provides a natural and elegant model for linear lambda-calculi like FUZZ, it cannot model standard (i.e. non-linear) typed lambda-calculi, since it is not cartesian closed. In this talk I will illustrate a few strategies to construct metric models of typed and untyped lambda-calculi, by either restricting to well-behaved classes of metric spaces, by relaxing the required Lipschitz conditions, or by considering alternative formulations of the usual metric axioms (as in Matthews’ partial metric spaces).

## Mathematics of Natural Language

This session was organized by Mehrnoosh Sadrzadeh and contained the following invited talks:

**Guy Emerson** (University of Cambridge) *Judicious Incoherence: Probabilistic Modelling Without A Joint Distribution*

There are many processes of language understanding that can be naturally related to each other in terms of Bayesian inference: for example, judging whether a description matches a scene, versus imagining a scene based on a description of it. However, it is known that Bayesian inference is intractable (in fact,  $\#\mathcal{P}$ -complete), even in seemingly simple settings. Assuming that a human mind cannot solve intractable problems in a split second, such cognitive processes cannot be coherent, in a strict Bayesian sense. I will sketch an approach to modelling cognitive processes as incoherent conditional distributions, without a joint distribution. Nonetheless, we can view coherence as an ideal (even if unreachable), and consider how far we must deviate from it. This allows us to explain some apparent inconsistencies in human behaviour as judiciously incoherent given our minds’ computational constraints.

**Tiffany Duneau** (Quantinuum, University of Oxford) *DisCoCirc: Theory and Experiments*

Natural language processing (NLP) has seen a huge boom with the success of large language models, however a common issue raised is their black box nature: when something goes wrong, we typically don’t understand why. DisCoCirc is a model that aims to provide an easier path towards interpretability by leveraging compositionality. This leads to two key notions: compositional generalisation - the capacity for the model to be trained on small instances of a problem, and then generalise to larger instances at test time - as well as compositional interpretability, by inspecting how the individual components of the model interact with each other in order to gain an understanding of the behaviour of the model as a whole.

**Daphne Wang** (Quandela, France) *Extracting structure from an LLM – how to improve on surprisal-based models of Human Language Processing*

Prediction and reanalysis are considered two key processes that underly humans’ capacity to comprehend language in real time. Computational models capture it using Large Language Models (LLMs) and a statistical measure known as ‘surprisal’. Despite successes of LLMs, surprisal-based models face challenges when it comes to sentences requiring reanalysis due to pervasive temporary structural ambiguities, such as garden path sentences. We ask whether structural information can be extracted from LLM’s and develop

a model that integrates it with their learnt statistics. When applied to a dataset of garden path sentences, the model achieved a significantly higher correlation with human reading times than surprisal. It also provided a better prediction of the garden path effect and could distinguish between sentence types with different levels of difficulty. duce disambiguation patterns for unseen phrases.

### Types and the Extraction of Correct Programs

This session was organized by Ulrich Berger and contained the following invited talks:

**Michal Konečný** (Aston University, Birmingham) *Verified Exact Real Computation using cAERN*

Exact real computation (ERC) offers a real number data type whose semantics matches real numbers as used in mathematics and physics, hiding from the programmer issues of accuracy loss due to rounding, making it easier to reason about the correctness of such programs. It is still desirable to formally verify ERC programs as they tend to be complex in subtle ways. Firstly, the representations of exact real numbers are either infinite objects or functions, and care is needed to avoid programs using excessive resources. Secondly, some ERC operations are partial (comparison, limit), and tasks often require non-deterministic branching. Our work focuses on the latter area of complexity with the help of computable analysis.

cAERN supports writing partial and nondeterministic ERC programs in Rocq, stating and verifying the functional properties, and extracting reasonably efficient Haskell programs that rely on the Haskell ERC library AERN. cAERN does not define or verify a concrete datatype of real numbers. Instead, it axiomatizes such a datatype as a computable version of an Archimedean ordered complete field and trusts the AERN real-number representation and its basic operations. I will present key cAERN concepts, illustrate them using examples, and compare cAERN with other approaches to verified ERC.

This is joint work with Holger Thies and Sewon Park.

**Fredrik Nordvall Forsberg** (University of Strathclyde) *Exponentiation of Ordinals in Homotopy Type Theory*

An important aspect of classical ordinals is their arithmetic theory, where addition, multiplication and exponentiation is generalised from natural numbers to the transfinite. Addition and multiplication can be defined using well-known and constructively non-problematic constructions, but exponentiation is usually defined using case distinctions on whether the exponent is zero, a successor or a limit – a definitional principle which is not available constructively. In this talk, I will instead present two seemingly different constructive definitions of ordinal exponentiation in the setting of homotopy type theory. The first is abstract, uses suprema of ordinals, and is solely motivated by the expected equations. The second is more concrete, based on decreasing lists, and can be seen as a constructive version of a classical construction by Sierpiński based on functions with finite support. A key result is that the two approaches are equivalent (whenever it makes sense to ask the question), and we can use this equivalence to prove algebraic laws and decidability properties of the exponential. We have formalised all our results in Agda. This talk is based on material that will appear at LICS 2025, and is joint work with Tom de Jong, Nicolai Kraus, and Chuangjie Xu.

**Ulrich Berger** (Swansea University) *Extracting concurrent programs*

We present a logical system CFP (Concurrent Fixed Point Logic) supporting the extraction of provably correct programs that permit a limited form of concurrency (two threads attempting concurrently to modify the same memory cell). We discuss two application in the area of exact real number computation: Conversion of infinite Gray code to signed digit representation, and Gaussian elimination. In both cases the extracted programs are nondeterministic. A noteworthy feature of CFP is the fact that its proof rules include a variant of the classical law of excluded middle. This is joint work with Hideki Tsuiki.